78/3

EISCAT
TECHNICAL
NOTES

THE EFFECT OF ICE ON AN ANTENNA REFLECTOR

T. Hagfors

KIRUNA
Sweden

EISCAT TECHNICAL NOTE No. 3
JANUARY 1978

THE EFFECT OF ICE ON AN ANTENNA REFLECTOR

T. Hagfors

Summary.
The effect of an ice layer on an antenna reflector is assessed by evaluating the phase deviation caused by an ice layer on a plane metallic reflector. The ice is treated as a slightly lassy dielectric. The calculations are made for an arbitrary angle of incidence and for both principal linear polarizations The results are presented in terms of curves of the phase of the reflections coefficients versus angle of incidence.

EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8764 GEOFYSK S

THE EFFECT OF ICE ON AN ANTENNA REFLECTOR

The presence of ice on an antenna reflector modifies the antenna performance in two ways. The phase of the reflected wave will deviate from the desired value; the reflection coefficient will be less than unity if there is appreciable loss in the layer. The former of these two effects is generally the more important since ice can often be treated as a slightly lossy dielectric.

To assess the effect we compute the reflection coefficient of a plane metallic reflector covered by a uniform layer of a lossy dielectric as shown in Figure 1.

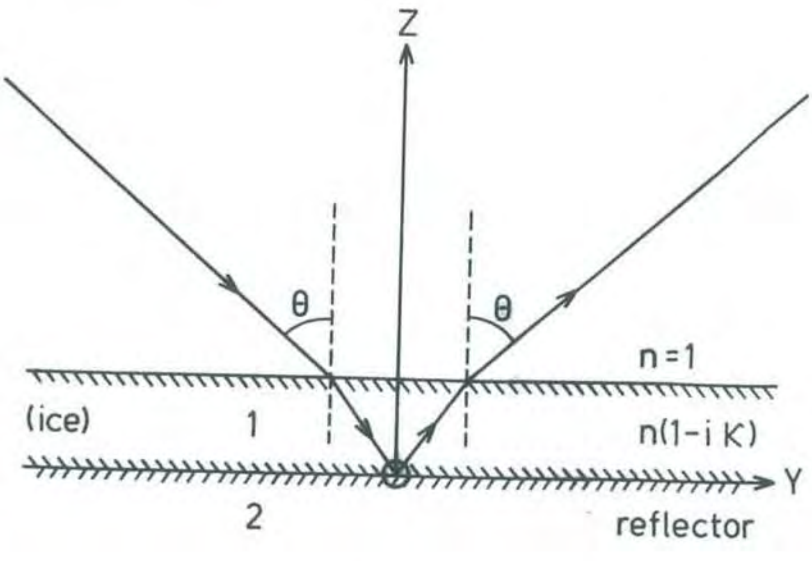

Figure 1. Lossy dielectric slab on perfect metallic reflector.

Above the dielectric slab we represent the field by

$$F_{0x} = A\, e^{-ik_0(y\cdot\sin\theta - z\cos\theta)} + B e^{-ik_0(y\sin\theta + z\cos\theta)} \quad (1)$$

where A, B signify the electric field normal to plane of incidence <u>or</u> the magnetic field in the x-direction when the polarization is orthogonal.

EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S

In the dielectric medium we have

$$F_{1x} = C \cdot e^{-ik_0 n(1-i\kappa)(y \cdot \sin\theta' - z\cos\theta')} + D \cdot e^{-ik_0 n(1-i\kappa)(y\sin\theta' + z\cos\theta')} \qquad (2)$$

In medium 2 the fields must vanish.

When A,B,C and D signify E fields, $\vec{E}$ is normal to the plane of incidence and when A,B,C and D signify B fields, $\vec{E}$ lies <u>in</u> the plane of incidence.

Since the y-variation must be identical in media (0) and (1) we must have:

$$n(1-i\kappa)\sin\theta' = \sin\theta \qquad (3)$$

We now put:

$$\theta' = \theta_1 + i\alpha \qquad (4)$$

and obtain:

$$(\sin\theta_1 \mathrm{Cosh}\alpha + i\cos\theta_1 \mathrm{Sinh}\alpha)n(1-i\kappa) = \sin\theta \qquad (5)$$

The two conditions must be satisfied:

A. $$-\kappa\sin\theta_1 \mathrm{Cosh}\alpha + \cos\theta_1 \mathrm{Sinh}\alpha = 0 \qquad (6)$$

which means that

$$\mathrm{Tgh}\alpha = \kappa \cdot \mathrm{tg}\theta_1 \qquad (7)$$

The argument α is proportional to κ to first order.

EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S

B. $n(\sin\theta_1 \text{Cosh}\alpha + \kappa\cos\theta_1 \cdot \text{Sinh}\alpha) = \sin\theta$

$$n\sin\theta_1 \cdot \text{Cosh}\alpha \cdot (1+\kappa^2) = \sin\theta \quad (8)$$

To first order in κ we recover Snell's law:

$$\frac{\sin\theta_1}{\sin\theta} = \frac{1}{n} \quad (9)$$

In the following boundary value consideration the y-variation can be ignored:

Case 1 E $\perp$ plane of incidence

$$\vec{E} = E_x \cdot \vec{e}_x \quad (10)$$

In medium 1 we must have $E_x = 0$ at $z = 0$. This gives:

$$C = -D. \quad (11)$$

At $z = d$ continuity of E_x gives:

$$C(e^{ik_0 n(1-i\kappa) z\cos\theta'} - e^{-ik_0 n(1-i\kappa) z\cos\theta'}) =$$

$$A \cdot e^{ik_0 z\cos\theta} + B \cdot e^{-ik_0 z\cos\theta} \quad \text{when } z = d. \quad (12)$$

Continuity of H gives:

$$C(n(1-i\kappa)\cos\theta') \cdot (e^{ik_0 n(1-i\kappa) z\cos\theta'} + e^{-ik_0 n(1-i\kappa) z\cos\theta'})$$

$$= \cos\theta(Ae^{ik_0 z\cos\theta} - Be^{-ik_0 z\cos\theta}) \quad \text{when } z = d. \quad (13)$$

Simplified system of equations for B and A:

$$A \cdot \eta_0 + B \cdot \eta_0^{-1} = C(\eta_1 - \eta_1^{-1})$$
$$A \cdot \eta_0 - B \cdot \eta_0^{-1} = C\frac{T}{S}(\eta_1 + \eta_1^{-1}) \quad (14)$$

EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S

$$S = \cos\theta$$
$$T = n(1-i\kappa)\cos\theta' \cong n(\cos\theta_1 - \frac{i\kappa}{\cos\theta_1})$$
$$\eta_0 = e^{ik_0dS}$$
$$\eta_1 = e^{ik_0dT}$$

From which we derive:

$$\frac{B}{A} = -\frac{G_-e^{ik_0dG_+} + G_+e^{-ik_0dG_-}}{G_+e^{ik_0dG_-} + G_-e^{-ik_0dG_+}} = R_\perp \qquad (15)$$

where (to first order in κ)

$$G_\pm = \sqrt{n^2-\sin^2\theta} \pm \cos\theta - \frac{i\kappa n^2}{\sqrt{n^2-\sin^2\theta}} \qquad (16)$$

Case 2. E || plane of incidence

$$\vec{B} = B_x \cdot \vec{e}_x \qquad \mu_1 = \mu_0 \qquad (17)$$

In medium 1 we must have $E_y = 0$ at $z = 0$. This means that

$$\frac{\partial B_x}{\partial z} = 0 \text{ at } z = 0 \qquad (18)$$

or $\quad C = D. \qquad (19)$

The two boundary conditions at z = d become:

$$A\cdot\eta_0 + B\cdot\eta_0^{-1} = C(\eta_1 + \eta_1^{-1})$$
$$A\cdot\eta_0 - B\cdot\eta_0^{-1} = \frac{C\cdot T}{S\cdot n^2(1-i\kappa)^2}(\eta_1-\eta_1^{-1}) \qquad (20)$$

Or:

$$R_{11} = \frac{H_-\cdot e^{ik_0dG_+} + H_+\cdot e^{-ik_0dG_-}}{H_+e^{ik_0dG_-} + H_-e^{-ik_0dG_+}} \qquad (21)$$

EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S

where G_+ and G_- were defined above and where to first order in κ we have:

$$H_{\pm} = n^2\cos\theta \pm \sqrt{n^2-\sin^2\theta} - i\kappa n^2\left(2\cos\theta \mp \frac{1}{\sqrt{n^2-\sin^2\theta}}\right) \quad (22)$$

Numerical example:

For an ice-layer κ is negligible and $n^2 \approx 3.0$, possibly slightly larger. Computations were made of $R_{\perp}$ and $R_{||}$ for several combinations of layer thickness and angles of incidence. Some results are shown in Figure 2.

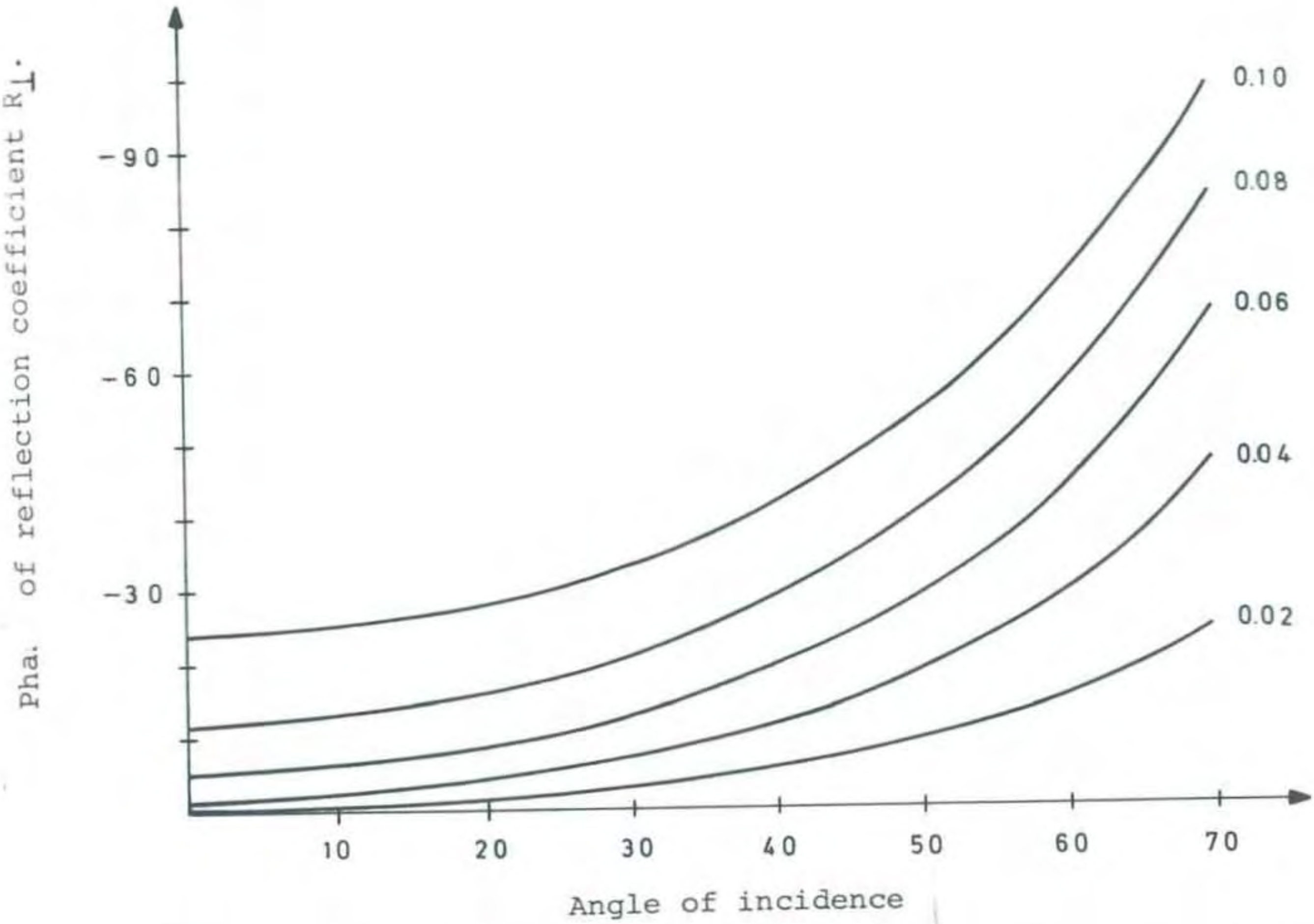

Figure 2.. Phase of reflection coefficient of $R_\perp$ (degrees) plotted against angle of incidence (degrees).

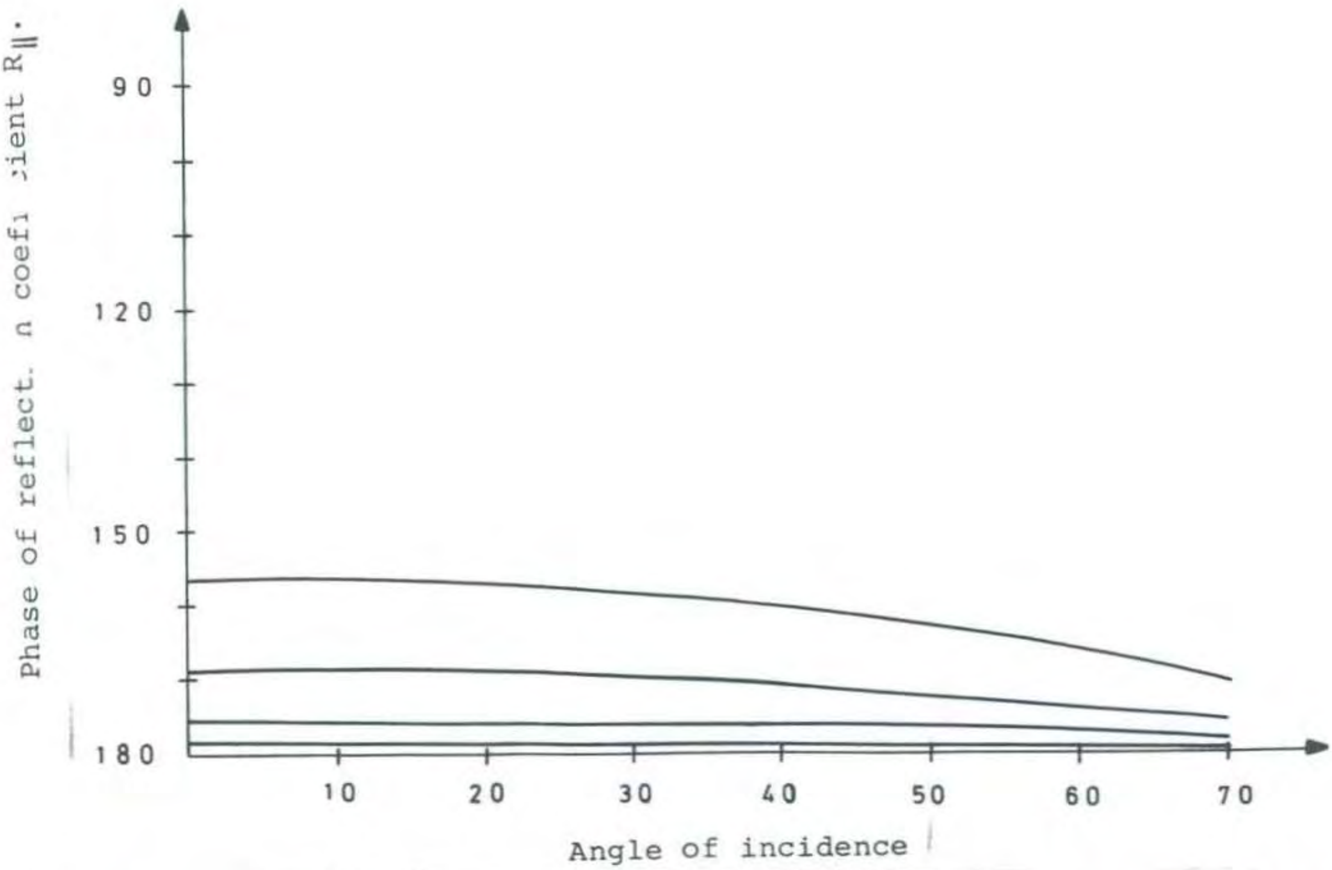

Figure 2B. Phase of reflection coefficient of $R_{/\!/}$ (degrees) plotted against angle of incidence (degrees).

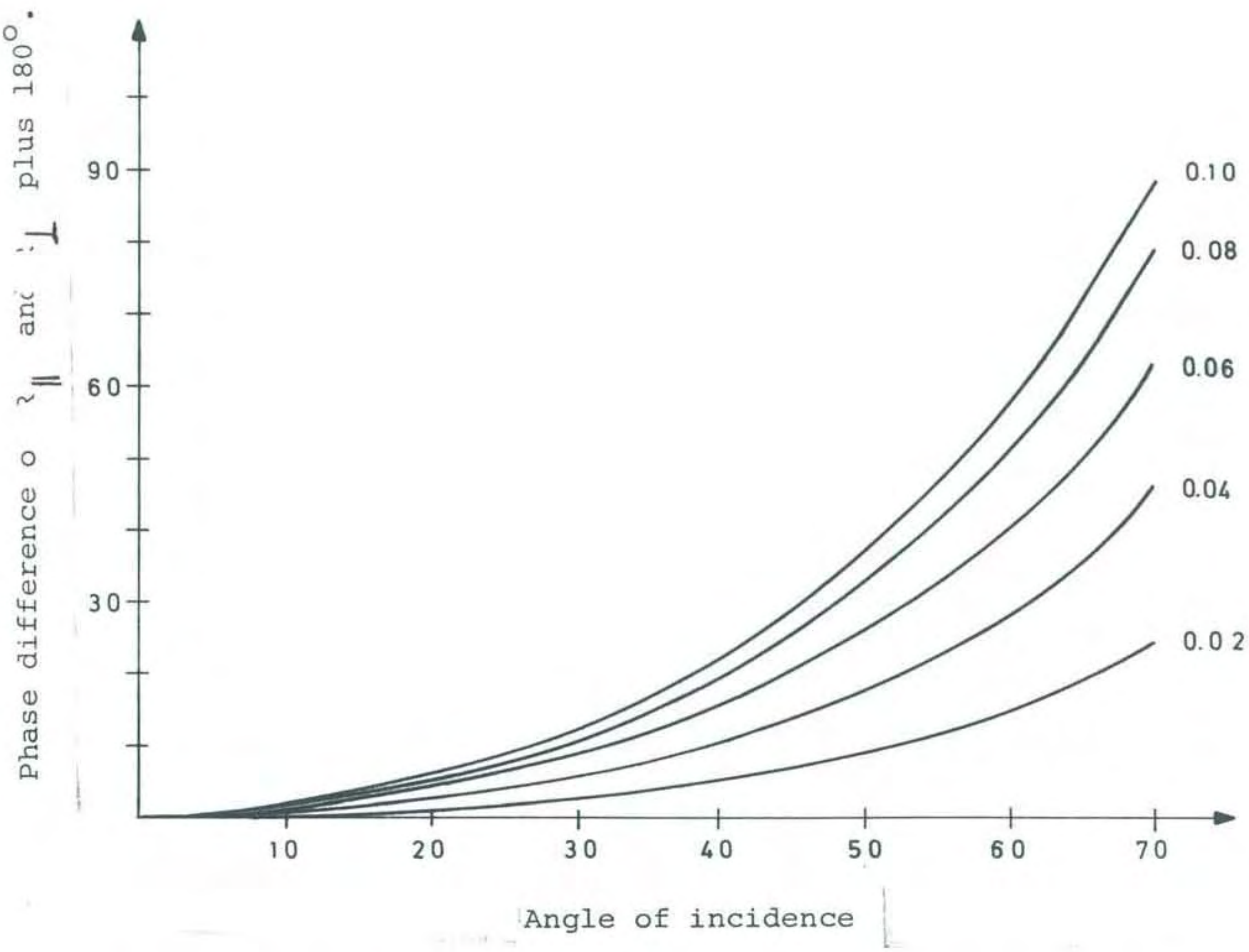

Figure 2C. Phase "error" of the phase difference between $R_{\parallel}$ and $R_{\perp}$ plotted against angle of incidence (degrees).

EISCAT SCIENTIFIC ASSOCIATION
S-98101 KIRUNA 1. SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S