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EISCAT
TECHNICAL
NOTES

DISCRETE PHASE STEERING BY PERMUTING PRECUT PHASE CABLES

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EISCAT TECHNICAL NOTE No 6/78
OCTOBER 1978.

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by

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Summary.

The note describes a method to phase steer a linear array antenna to a discrete number of direction by rearranging the sequence of phase cables that are connected to each array element. The phase cables must be precut to specific lengths that are evaluated.

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1. Introduction.

In May 1978 a cheap method to phase steer the EISCAT VHF cylinder antenna was proposed by the author. The method uses phase cables that are connected to each array element. By permuting these phase cables, e.g. disconnect them and then connect them to another element, in special ways, it is possible to scan the beam to a large number of directions. A simple example illustrates the idea.

Consider a linear array of N=6 elements with spacing $d=0.5\lambda$, and $N_c=7$ phase cables with lengths equal to a constant plus

$$\ell_i = \frac{i}{7}\lambda_0 \qquad i = 0, 1, \ldots, 6$$

where λ_0 is the wavelength at the center frequency, and ℓ_i is the length of cable number i. The sequences of cables along the array and the corresponding scan angles are shown in the table below. The six array elements are numbered successively j = 1, 2, ..., 6.

Array element number:		1	2	3	4	5	6	Scan angle $\theta_n=\arcsin(\frac{n}{7}\frac{\lambda_0}{d})$
Sequence of phase cables	for n=1 :	$\frac{0}{7}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{4}$	$\frac{5}{7}$	θ_1 = 16.6 deg
	for n=2 :	$\frac{0}{7}$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{6}{7}$	$\frac{1}{7}$	$\frac{3}{7}$	θ_2 = 34.8 deg
	for n=3 :	$\frac{0}{7}$	$\frac{3}{7}$	$\frac{6}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	$\frac{1}{7}$	θ_3 = 59.0 deg

The table shows scan angles on one side of broadside. The same scan angles on the other side of broadside are, of course, avaiable in the same way, but with the array elements numbered in the opposite direction.

In this array it was possible to reach three scan angles on either side of broadside with 7 phase cables only. That would have been possible in a 7 elements array as well.

In larger arrays more scan angles may be reached in a similar way.

2. Optimum Choice of Cable Lengts.

The possible scan angles for a linear array with N elements is found as follows, using N_c phase cables.

Let $N_c \geq N$. The N_c phase cables are chosen with lengths equal to a constant plus

$$\ell_i = \frac{i}{N_c}\lambda_0 \qquad i = 0, 1, \ldots, N_c-1 \tag{1}$$

Thus, the phase of cable number i is

$$\phi_i = 2\pi\frac{i}{N_c} \qquad i = 0, 1, \ldots., N_c-1 \tag{2}$$

The phase progression from one array element to the next is assumed to be

$$\Delta\phi = 2\pi\frac{n}{N_c} \tag{3}$$

where n is an integer and $n < N_c$. The scan angle from broadside will then be

$$\theta_n = \arcsin\left(\frac{\Delta\phi\lambda_0}{2\pi d}\right) = \arcsin\left(\frac{n}{N_c}\frac{\lambda_0}{d}\right) \tag{4}$$

where d is the element spacing.

Let the elements in the array be numbered successively as $j = 1, 2, \ldots, N$. If the scan angle in Eq. (4) is to be reached, the phase excitation ϕ_j of the array elements must give the phase progression in Eq. (3). Thus

$$\phi_j = 2\pi\frac{n\,j}{N_c} \qquad j = 1, 2, \ldots, N \tag{5}$$

One condition for the phase cables in Eq. (1) and (2) to be used to obtain the phase excitation ϕ_j, is that

$$0 \leq \phi_j < 2\pi \tag{6}$$

Therefore,

$$\phi_j = 2\pi\left(\frac{n\,j}{N_c} - \mathrm{Int}\left(\frac{n\,j}{N_c}\right)\right) \qquad j = 1, 2, \ldots, N \tag{7}$$

where Int(x) is a function taking the integer part of x. The phase excitation in Eq. (7) gives the same scan angle at the center frequency as the excitation in Eq. (5).

It is evident that for any array element j_1 in Eq. (7) it is possible to find a phase cable i_1 in Eq. (2) so that $\phi_{i_1} = \phi_{j_1}$. But there is only <u>one</u> cable i_1 with phase ϕ_{i_1}, so that any other array element $j_2 (j_2 \neq j_1)$ must have a phase excitation $\phi_{j_1} \neq \phi_{i_1}$. In other words, the phase excitation in Eq. (7) is possible with the phase cables in Eq. (1) if Eq. (7) never repeats any of the cable phases in Eq. (2). Thus, for any two array elements j_1 and j_2, $j_2 \neq j_1$

$$\phi_{j_1} \neq \phi_{j_1} \tag{8}$$

It is convenient to choose $j_2 > j_1$. Eq. (7) gives

$$2\pi\left(\frac{nj_1}{N_c} - \mathrm{Int}\left(\frac{nj_1}{N_c}\right)\right) \neq 2\pi\left(\frac{nj_2}{N_c} - \mathrm{Int}\left(\frac{nj_2}{N_c}\right)\right)$$

or

$$\frac{n(j_2-j_1)}{N_c} \neq \mathrm{Int}\left(\frac{nj_2}{N_c}\right) - \mathrm{Int}\left(\frac{nj_1}{N_c}\right) \tag{9}$$

This equation is satisfied if and only if

$$\frac{n(j_2-j_1)}{N_c} \neq I \tag{10}$$

where I is any integer number and $(j_2-j_1) = 1, 2, \ldots, N-1$.

Consider two cases:

a) N_c is a prime number. Eq. (10) is then always satisfied because $n < N_c$ and $j_2-j_1 \leq N-1 < N_c$.

b) N_c is not a prime number. Let N_c be factorized in its prime factors P_k, $k = 1, 2, \ldots, K$.

$$N_c = \prod_{k=1}^{K} P_k \qquad (11)$$

Let also N_c be factorized in any two integer factors F_1 and F_2 , e.g.

$$N_c = F_1 F_2 \qquad (12)$$

Eq. (10) is not satisfied when

$$n = \ell' F_1 \qquad \ell' = 1,\ 2,\ \ldots,\ \frac{N_c}{F_1} - 1 \qquad (13)$$

because when $j_2 - j_1 = F_2$

$$\frac{n(j_2 - j_1)}{N_c} = \frac{\ell' F_1 F_2}{N_c} = \ell' = I \qquad (14)$$

Thus, because $j_2 - j_1 \leq N - 1 < N_c$, Eq. (10) is satisfied when n has no common factors with N_c , e.g.

$$n \neq \ell' F_1 \qquad \ell' = 1,\ 2,\ \ldots,\ \frac{N_c}{F_1} - 1 \qquad (15)$$

where F_1 is any factor in N_c. This is alternatively written

$$n \neq \ell P_k \qquad \ell = 1,\ 2,\ \ldots,\ \frac{N_c}{P_k} - 1 \qquad (16)$$

where P_k is any prime factor in N_c and $k = 1,\ 2,\ \ldots,\ K$. The numbers $\ell' F_1$ in Eq. (15) are included in the numbers ℓP_k in Eq. (16) when P_k is a factor in F_1 , because $\ell P_k = F_1$ when $\ell = \frac{F_1}{P_k}$.

The conclusion is as follows: Phase steering with the N_c phase cables given in Eq. (2) is possible to scan angles θ_n in Eq. (4), if n is any integer less than N_c that is not a multiple of any prime factor in N_c , see Eq. (16). To be able to scan the beam to as many directions θ_n as possible, it is necessary, therefore, to choose N_c as a prime number, or with high valued prime factors.

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