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NOTES

BALANCE BETWEEN INVESTMENT IN REFLECTOR AND FEED IN THE VHF CYLINDRICAL ANTENNA

P.S. Kildal and T. Hagfors

KIRUNA
Sweden

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Summary.

The note describes a method for evaluating the cost of the feeder system in such a way that the feed-investment is as cost effective as the investment in the mechanical structure.

1. OPTIMIZATION OF VHF ANTENNA PERFORMANCE.

The Scientific requirements of EISCAT dictates that the VHF antenna configuration be a cylindrical structure which can be tilted mechanically in one plane and which can be phased into two different directions in the other plane.

In such a structure the feeder system constitutes a substantial part of the cost, particularly when the cylinder is very long. It therefore becomes important to strike the correct balance between the cost of the feeder and RF distribution system on the one side and the reflector structure on the other.

The criterion used for the optimization procedure will here be taken to be the ratio of signal power to noise power at the receiver input. In incoherent scatter observations where the volume of scattering is determined by the cross section of the antenna beam, the relevant quality factor K is given by:

$$K = \frac{(A_{eff})^2}{T_{eff}} \quad \frac{1}{\eta_b \eta_a \cdot A} \qquad (1)$$

where: A_{eff} = effective transmit/receive antenna aperture

T_{eff} = effective antenna temperature

Here we have:

$$A_{eff} = \eta_t \eta_a \eta_b \cdot A \qquad (2)$$

where: A = physical antenna aperture

η_t = efficiency due to transmission line loss

η_a = efficiency due to non-ideal illumination

η_b = efficiency due to feeder blockage

The effective noise temperature may be expressed as:

$$T_{eff} = \eta_t \cdot T_a + (1-\eta_t)T_0 + T_{rec} \tag{3}$$

where: T_0 = temperature of transmission line

T_{rec} = noise temperature of receiver

T_a = antenna temperature

The antenna temperature T_a may be expressed as:

$$T_a = (1-\eta_{sp})T_{sky} + \eta_{sp}T_{ground} \tag{4}$$

where:

T_{sky} = the sky temperature

T_{ground} = ground temperature

η_{sp} = fraction of antenna power coupled to ground

η_{sp} depends on the direction of beam pointing and on the aperture illumination of the antenna. The aperture illumination also determines η_a .

The sky temperature at a frequency of 224 MHz varies between 120 and $1200^{\circ}K$(1) depending on the direction of the beam in relation to the galactic coordinate system.

The quality factor appropriate to the VHF antenna when used for incoherent scatter observations hence becomes:

$$K = \frac{\eta_t^2 \eta_a \eta_b \cdot A}{\eta_t T_a + (1-\eta_t)T_t + T_{rec}} \tag{5}$$

An optimally designed antenna for the present purpose must be designed to optimize K subject to the constraints which must be imposed. Below we shall apply this philosophy in order to strike a balance between the investments in reflector and in feeder system. The constraint will be a given total antenna system price. We also limit the discussion to a cylindrical reflector antenna which appears to be the only one which can satisfy all the requirements of the performance specifications.

2. OPTIMIZATION OF CYLINDRICAL ANTENNA PERFORMANCE.

The parabolic cylinder antenna will be assumed to be divided into N identical sections, each of width D and of length L. The total cylinder length hence will be NL.

We shall assume that the feeder elements are of a specific given type so that the only permissible variations allowed will be in the size of the reflector area and in the transmission line. We shall, in particular assume that the aperture efficiency η_a remains constant when parameter variation occurs.

2.1. Functional Dependence of the Quantities in the Quality Factor.

2.1.1. Aperture area.

$$A = L \cdot D \cdot N$$

The physical area is somewhat larger, but we assume the ratio of focal length to antenna width to be given. This means that the aperture width D and the physical area are proportional. It also means that the feeder elements and therefore η_a remain the same as D varies.

2.1.2. Blockage efficiency.

The blockage efficiency η_b is a function of L and D. To a first approximation one can assume the blockage area to be constant and not vary with D in which case:

$$\eta_b(D) \approx \eta_{b0} - (1-\eta_{b0})\frac{D-D_c}{D_0} \qquad (6)$$

where D_0 is a standard antenna width.

2.1.3. Transmission line efficiency.

The transmission line efficiency η_t is a function of antenna length L, number of sections N and of the type of transmission line used. We shall use η_t as a free variable below.

2.1.4. Antenna temperature.

The antenna temperature depends jointly on sky noise and antenna as in (4). The sky temperature depends on the beam pointing in relation to the galactic coordinate system. The spillover efficiency η_{sp} depends on the antenna pointing relative to the local environment.

2.1.5. Transmission line temperature.

The transmission line temperature T_0 is primarily determined by the environmental temperature. Deviations may be caused by transmitter power P_t coupled with line losses, hence T_0 depends on both P_t and η_t in a complicated manner. We shall assume it to be constant near the environmental value.

2.1.6. Receiver temperature.

The effective receiver temperature is entirely determined by the quality of the receiver and does not depend on η_t or on D.

2.1.7. Cost of Mechanical structure.

The mechanical cost depends primarily on L, D and N, i.e.

$$C_m = C_m(L, D, N)$$

2.1.8. Cost of transmission line and feeder elements.

The price of the transmission line depends strongly on η_t , L and N and must be dimensioned to take the transmitter power P_t . The important point here is that the transmission line cost is independent of the width D.

2.2. Cost Balance between Width and Feeder Losses.

We now attempt to make K a maximum subject to variations in D and η_t under the constraint that the total price be given, i.e.:

$$C = C_m + C_t = \text{constant} \qquad (7)$$

This means that:

$$\delta C = 0 = \delta C_m + \delta C_t \tag{8}$$

Since we assume that $C_m = C_m(D)$ and

$$C_t = C_t(\eta_t)$$

we have:

$$\delta C_m = \left(\frac{dC_m}{dD}\right)_{D_0} \cdot \delta D \tag{9}$$

and:

$$\delta C_t = \left(\frac{dC_t}{d\eta_t}\right) \cdot \delta \eta_t \tag{10}$$

hence $$\frac{\delta\eta_t}{\delta D} = - \left(\frac{dC_m}{dD}\right)_{D_0} / \left(\frac{dC_t}{d\eta_t}\right)_{1_t}$$

A change in η_t and D subject to the above constraint is advantageous as long as the quality factor increases, or $\delta K > 0$.

From (5) we have:

$$\delta K = \left(\frac{\partial K}{\partial D}\right) \delta D + \left(\frac{\partial K}{\partial \eta_t}\right) \delta \eta_t \tag{11}$$

where:

$$\left(\frac{\partial K}{\partial D}\right)_{D_0} = \frac{\eta_t^2 \eta_a}{\eta_t \cdot T_a + (1-\eta_t) T_0 + T_{rec}} \eta_b \cdot N \cdot L - N \cdot L \cdot D_0 \cdot (1-\eta_{b0}) \frac{1}{D_0}$$

$$\approx K \cdot \frac{2-\eta_b^{-1}}{D} \tag{12}$$

and:

$$\left(\frac{\partial K}{\partial \eta_t}\right)_{\eta_t} = K \frac{2}{\eta_t} - \frac{T_a - T_0}{\eta_t T_a + (1-\eta_t) T_0 + T_{rec}} \tag{13}$$

Hence:

$$\frac{\delta K}{K} = \left(2 - \frac{1}{\eta_b}\right) \frac{\delta D}{D} + \left(2 - \frac{T_a - T_0}{T_a - T_0 + \frac{T_0 + T_{rec}}{\eta_t}}\right) \frac{\delta \eta_t}{\eta_t}$$

A profitable investment in the feeder system can, therefore, be made when:

$$\left(2 - \frac{T_a - T_0}{T_a - T_0 + \frac{T_0 + T_{rec}}{\eta_t}}\right)\frac{\delta\eta_t}{\eta_t} > \left(2 - \frac{1}{\eta_b}\right)\frac{\delta C_t}{D_0\left(\frac{dC_m}{dD}\right)} \qquad (14)$$

or:

$$\delta C_t < \frac{\delta\eta_t}{\eta_t} \cdot \frac{2}{\left(2 - \frac{1}{\eta_b}\right)} \cdot D_0\left(\frac{dC_m}{dD}\right)_{D_0} \cdot \underbrace{\frac{1+0.5\nu}{1+\nu}}_{S} \qquad (15)$$

where:

$$\nu = \eta_t \frac{T_a - T_0}{T_a + T_{rec}}$$

Transmission line losses are usually expressed in decibles as:

$$D = 10 \log_{10} \frac{1}{\eta_t}$$

For an improved line with better efficiency $\eta_t + \delta\eta_t$ the corresponding decibel loss is:

$$D - \Delta = 10 \log_{10} \frac{1}{\eta_t + \delta\eta_t}$$

The improvement of one line over the other measured in decibles becomes:

$$\Delta = 10 \log_{10} \frac{\eta_t + \delta\eta_t}{\eta_t} = 10 \log_{10}\left(1 + \frac{\delta\eta_t}{\eta_t}\right)$$

$$\approx \frac{10}{2.3}\left(\frac{\delta\eta_t}{\eta_t} - \frac{1}{2}\left(\frac{\delta\eta_t}{\eta_t}\right)^2 \ldots\right)$$

Hence, to first order:

$$\frac{\delta\eta_t}{\eta_t} \approx 0.23\Delta$$

2.3. Numerical Example.

Numerically for the EISCAT VHF Cylindrical antenna operating at 224 MHz one may assume (2):

$$D_0 \left(\frac{dC_m}{dD}\right)_{40m} \gtrapprox 17 \cdot 10^6 \text{Swedish kr.}$$

$$T_{amin} = 125^{o}K$$

$$T_{amax} = 1200^{o}K$$

$$T_0 = 290^{o}K$$

$$T_{rec} = 120^{o}K$$

Because of the offset feed design one may put the blockage efficiency equal to unity:

$$\eta_b = 1$$

With a typical value of η_t of 0.9 the quantity S will vary between 1.35 (at T_{amin}) and 0.68 (at T_{amax})

From this it follows that

$$\delta C_t < 10.6 \cdot \Delta \text{ MSw.kr. when } T_a = 125^{o}K$$

$$\delta C_t < 5.3 \cdot \Delta \text{ MSw.kr. when } T_a = 1200^{o}K$$

Note that Δ is express in decibel line improvement associated with the cost increase δC_t and that the price increase encompasses the sum of both polarizations.

References:

(1) Brown, R.L. "Galactic Non-Thormal Continuum Emission" in "Galactic and Extra-Galactic Radio Astronomy" edited by Verschuur, G.L. and K.I. Kellermann. Springer-Verlag Berlin, Heidelberg, New York (1974).

(2) Myklebust, A. and P. Skjæveland, "Bærekonstruksjon for VHF antenna. Alternativ: Usymmetrisk vugge. Trondheim, August 12, (1977).

EISCAT SCIENTIFIC ASSOCIATION
S-981 01 KIRUNA 1, SWEDEN
TELEPHONE 0980/187 40
TELEX 8754 GEOFYSK S